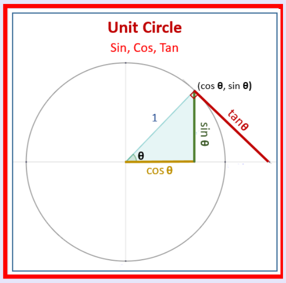
Math 241

Winter 2023

Lecture 3



Class QZ 2

1) Simplify $(2x-5)(4x^2+10x+25)$

$= 8x^3 + \cancel{20x^2} \cancel{+50x} \cancel{-20x^2} \cancel{-50x} - 125$

$= \boxed{8x^3 - 125}$ ✓

2) Factor Completely $x^2 - 12x + 36$

Lead. Coef.
1

$= (x-6)(x-6)$

$= \boxed{(x-6)^2}$

Repeated Factor

-1, -36
-2, -18
-3, -12
-4, -9
$\boxed{-6, -6}$

Square - Root Method:

If $x^2 = K$, then $x = \pm\sqrt{K}$

Solve $x^2 = 9$ by S.R.M.

$x = \pm\sqrt{9}$ $\boxed{x = \pm 3}$

Solution Set

$\{\pm 3\}$

Solve

$(2x-1)^2 - 3 = 4$

$(2x-1)^2 = 7$

use S.R.M.

$2x-1 = \pm\sqrt{7}$

$2x = 1 \pm \sqrt{7}$

$\boxed{x = \frac{1 \pm \sqrt{7}}{2}}$

$\left\{\frac{1 \pm \sqrt{7}}{2}\right\}$

Solve $(3x+2)^2 + 5 = 30$

$(3x+2)^2 = 25$

use S.R.M.

$3x+2 = \pm\sqrt{25}$

$3x+2 = \pm 5$

$3x = -2 \pm 5$

$x = \frac{-2 \pm 5}{3}$

$x = \frac{-2+5}{3}$

$= \frac{3}{3} = \boxed{1}$

$x = \frac{-2-5}{3}$

$= \boxed{\frac{-7}{3}}$

$\left\{\frac{-7}{3}, 1\right\}$

Solving $x^2 + bx + C = 0$ by Completing the Square method:

L.C. = 1

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -C + \left(\frac{b}{2}\right)^2$$

Take half of b, Square it, and add to both Sides

$$x^2 + bx + \frac{b^2}{4} = \frac{b^2}{4} - \frac{C \cdot 4}{4}$$

$$\left(x + \frac{b}{2}\right)^2 = \frac{b^2 - 4C}{4}$$

use S.R.M.

$$x + \frac{b}{2} = \pm\sqrt{\frac{b^2-4C}{4}}$$

$$x = \frac{-b}{2} \pm \frac{\sqrt{b^2-4C}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2-4C}}{2}$$

Solve $x^2 + 8x + 12 = 0$ by completing the Square method.

$$x^2 + 8x + 16 = -12 + 16$$

$$\frac{1}{2} \cdot 8 = 4$$

$$4^2 = 16$$

$$(x+4)^2 = 4$$

Now use S.R.M.

$$x + 4 = \pm\sqrt{4}$$

$$x + 4 = \pm 2$$

$$x = -4 \pm 2$$

$$x = -4 + 2 = \boxed{-2}$$

$$x = -4 - 2 = \boxed{-6}$$

$$\{-6, -2\}$$

Solve $x^2-10x+25=0$ by Completing the Square method.

$$\underbrace{x^2-10x+25}=-25+25$$

$\frac{1}{2}\cdot(-10)=-5$

$(-5)^2=25$

$$(x-5)^2=0$$

by S.R.M.

$$x-5=\pm\sqrt{0}$$

$$x=5\pm0$$

$$\boxed{x=5}$$

$\{5\}$

Repeated Solution

Solve $x^2-3x-28=0$ by Completing the Square method.

$$\underbrace{x^2-3x+\frac{9}{4}}=\underbrace{28+\frac{9}{4}}$$

$\frac{1}{2}\cdot(-3)=\frac{-3}{2}$

$\left(\frac{-3}{2}\right)^2=\frac{9}{4}$

$$\left(x-\frac{3}{2}\right)^2=\frac{28\cdot4}{4}+\frac{9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=\frac{112+9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=\frac{121}{4}$$

Use S.R.M.

$$x-\frac{3}{2}=\pm\sqrt{\frac{121}{4}}$$

$$x=\frac{3}{2}\pm\frac{11}{2}$$

$$x=\frac{3+11}{2}=\frac{14}{2}=\boxed{7}$$

$$x=\frac{3-11}{2}=\frac{-8}{2}=\boxed{-4}$$

$\{-4, 7\}$

Let's derive the quadratic formula:

Quadratic Equation $ax^2+bx+c=0,\ a\neq 0$

Quadratic Formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$ax^2+bx+c=0$

$ax^2+bx=-c$

Divide by a

$\frac{a}{a}x^2+\frac{b}{a}x=-\frac{c}{a}$

$\underbrace{x^2+\frac{b}{a}x+\frac{b^2}{4a^2}}=-\frac{c}{a}+\frac{b^2}{4a^2}$

Take half of $\frac{b}{a}$

$\frac{1}{2}\cdot\frac{b}{a}=\frac{b}{2a}$

Square it

$\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$

Add this to both sides

$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c\cdot 4a}{a\cdot 4a}$

$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{4ac}{4a^2}$

$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$

by S.R.M.

$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$

$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Solve $(2x+5)(3x-2)=7$ by using the quadratic formula.

Hint:

Foil, Simplify, write in $ax^2+bx+c=0$ form.

$6x^2-4x+15x-10-7=0$

$6x^2+11x-17=0 \Rightarrow a=6 \quad b=11 \quad c=-17$

$ax^2+bx+c=0$

$b^2-4ac=11^2-4(6)(-17)=529$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-11\pm\sqrt{529}}{2(6)}=\dfrac{-11\pm 23}{12}$

$x=\dfrac{-11+23}{12}=\dfrac{12}{12}=\boxed{1}$

$x=\dfrac{-11-23}{12}=\dfrac{-34}{12}=\boxed{\dfrac{-17}{6}}$

$\left\{-\frac{17}{6},1\right\}$

Solve $(\sqrt{5}\,x - 2)(\sqrt{5}\,x + 2) = 0$

Use Zero - Product Rule

If $A \cdot B = 0$, then $A = 0$ or $B = 0$ (Maybe both)

$\sqrt{5}\,x - 2 = 0$ OR $\sqrt{5}\,x + 2 = 0$

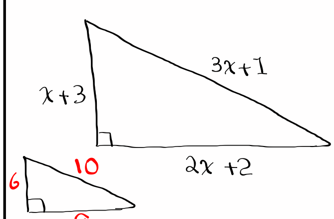
$\sqrt{5}\,x = 2$ $\qquad$ $\sqrt{5}\,x = -2$

$x = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ $\qquad$ $x = \frac{-2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$x = \frac{2\sqrt{5}}{5}$ $\qquad$ $x = \frac{-2\sqrt{5}}{5}$

$\left\{ \pm \frac{2\sqrt{5}}{5} \right\}$

Find all three sides:



Right Triangle

Pythagorean thrm

$a^2 + b^2 = c^2$

$(x+3)^2 + (2x+2)^2 = (3x+1)^2$

$x^2 + 6x + 9 + 4x^2 + 8x + 4 = 9x^2 + 6x + 1$

$5x^2 + 14x + 13 = 9x^2 + 6x + 1$

$9x^2 + 6x + 1 - 5x^2 - 14x - 13 = 0$

$4x^2 - 8x - 12 = 0$

Divide by 4

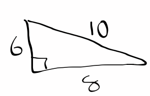
$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

by Z.P.R.

$x - 3 = 0$ OR $x + 1 = 0$

$\boxed{x = 3}$ $\qquad$ $\cancel{x = -1}$

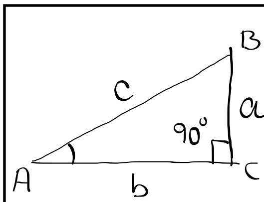


$6^2 + 8^2 = 10^2$

$36 + 64 = 100$

$100 = 100$ ✓

Three Sides are 6, 8, and 10.



$\sin A = \frac{a}{c}$ $\quad \csc A = \frac{c}{a}$

$\cos A = \frac{b}{c}$ $\quad \sec A = \frac{c}{b}$

$\tan A = \frac{a}{b}$ $\quad \cot A = \frac{b}{a}$

$\sin^2 A + \cos^2 A = 1$

$1 + \tan^2 A = \sec^2 A$

$1 + \cot^2 A = \csc^2 A$

$\csc A = \frac{1}{\sin A}$ $\quad \tan A = \frac{\sin A}{\cos A}$

$\sec A = \frac{1}{\cos A}$

$\cot A = \frac{1}{\tan A}$ $\quad \cot A = \frac{\cos A}{\sin A}$

$\sin(-A) = -\sin A$ $\quad \csc(-A) = -\csc A$

$\cos(-A) = \cos A$ $\quad \sec(-A) = \sec A$

$\tan(-A) = -\tan A$ $\quad \cot(-A) = -\cot A$

Special angles: $30°, 45°, 60°$

$\sin 30° = \frac{1}{2}$ $\quad \sin 45° = \frac{\sqrt{2}}{2}$ $\quad \sin 60° = \frac{\sqrt{3}}{2}$

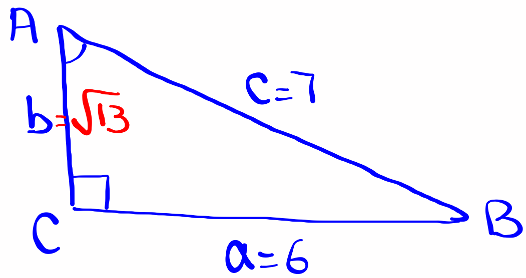
$\cos 30° = \frac{\sqrt{3}}{2}$ $\quad \cos 45° = \frac{\sqrt{2}}{2}$ $\quad \cos 60° = \frac{1}{2}$

$\tan 30° = \frac{\sqrt{3}}{3}$ $\quad \tan 45° = 1$ $\quad \tan 60° = \sqrt{3}$

Use your calc to find the following, round to 3-decimal places.

1) $\sin 25° \approx .423$ $\quad$ 2) $\cos 65° \approx .423$ $\quad$ 3) $\tan 40° \approx .839$

Find all six trig. functions for angle A when $a = 6$ and $c = 7$ in right triangle ABC.



$a^2 + b^2 = c^2$

$6^2 + b^2 = 7^2$

$36 + b^2 = 49$

$b^2 = 13$ $\quad b = \sqrt{13}$

$\sin A = \frac{6}{7}$

$\cos A = \frac{\sqrt{13}}{7}$

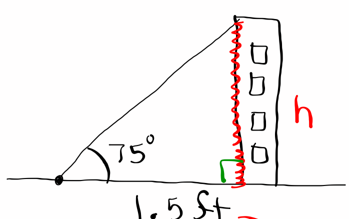
$\tan A = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$

$\csc A = \frac{7}{6}$

$\sec A = \frac{7}{\sqrt{13}} = \frac{7\sqrt{13}}{13}$

$\cot A = \frac{\sqrt{13}}{6}$

The angle of elevation to the top of a building is 75° from a point 1.5 ft away from the building. How tall is the building? Drawing Required.



1-decimal

$\tan 75° = \frac{h}{1.5}$

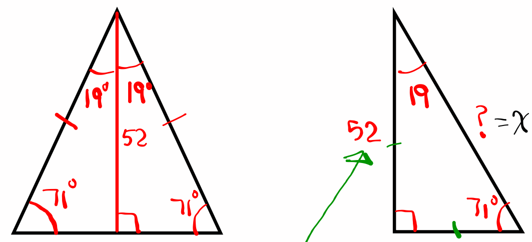
Cross-Multiply

$h = 1.5 \cdot \tan 75°$

$h = 5.59807\ldots$

$h \approx 5.6$ ft

The height of an isosceles triangle is 52 cm and each of the two equal angles are 71°. Find the equal Sides. Drawing required.



$x \cdot \cos 19° = 52$

$x = \dfrac{52}{\cos 19°}$

$x \approx 54.99627....$

$x \approx 55\,cm$

$\sin 71° = \dfrac{52}{x}$

whole

$\cos 19° = \dfrac{52}{x}$

Round to whole

equal Sides are 55 cm each

Verify $(\sin\alpha + 1)(\sin\alpha - 1) = -\dfrac{1}{\sec^2\alpha}$ ✓

$(A + B)(A - B)$

$= A^2 - B^2$

$(\sin\alpha + 1)(\sin\alpha - 1) = \boxed{\sin^2\alpha} - 1$

but $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$= \cancel{1} - \cos^2\alpha \cancel{-1}$

$= -\cos^2\alpha$

$= -\left(\dfrac{1}{\sec\alpha}\right)^2$

$= -\dfrac{1}{\sec^2\alpha}$ ✓

Verify $\dfrac{\cos\alpha}{\sec\alpha} + \dfrac{\sin\alpha}{\csc\alpha} = 1$ ✓

$$\frac{\cos\alpha}{\sec\alpha} + \frac{\sin\alpha}{\csc\alpha} = \cos\alpha \cdot \frac{1}{\sec\alpha} + \sin\alpha \cdot \frac{1}{\csc\alpha}$$

$$\frac{3}{5} = 3 \cdot \frac{1}{5}$$

$$\frac{\cos\alpha}{\frac{1}{\cos\alpha}} = \cos\alpha \div \frac{1}{\cos\alpha}$$

$$= \cos\alpha \cdot \cos\alpha$$

$$= \cos^2\alpha$$

$$= \cos\alpha \cdot \cos\alpha + \sin\alpha \cdot \sin\alpha$$

$$= \cos^2\alpha + \sin^2\alpha$$

$$= 1 ✓$$

Verify $\dfrac{\cos^4 x - \sin^4 x}{\sin^2 x} = \cot^2 x - 1$ ✓

$$A^4 - B^4 = (A^2)^2 - (B^2)^2 = (A^2 + B^2)(A^2 - B^2)$$

$$\frac{\cos^4 x - \sin^4 x}{\sin^2 x} = \frac{\overbrace{(\cos^2 x + \sin^2 x)}^{1}(\cos^2 x - \sin^2 x)}{\sin^2 x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} - \boxed{\frac{\sin^2 x}{\sin^2 x}}$$

$$\frac{A-B}{B} = \frac{A}{B} - \frac{B}{B}$$

$$= \left(\frac{\cos x}{\sin x}\right)^2 - 1$$

$$= \boxed{\cot^2 x - 1} ✓$$

Verify $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x} = 2\csc^2 x$ ✓ ✓

$$\frac{1}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x} + \frac{1}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x} =$$

$$\frac{1-\cancel{\cos x}+1+\cancel{\cos x}}{(1+\cos x)(1-\cos x)} = \frac{2}{(1+\cos x)(1-\cos x)}$$

$(A+B)(A-B) =$
$A^2 - B^2$

$$= \frac{2}{1-\cos^2 x} = \frac{2}{\sin^2 x}$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$$= 2\cdot\frac{1}{\sin^2 x} = 2\cdot\csc^2 x$$

$= 2\csc^2 x$ ✓

Verify $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = 2\sec x$

$$\frac{\cos x}{1+\sin x}\cdot\frac{\cos x}{\cos x} + \frac{1+\sin x}{\cos x}\cdot\frac{1+\sin x}{1+\sin x} =$$

$$\frac{\cos^2 x + (1+\sin x)(1+\sin x)}{(1+\sin x)\cdot\cos x} = \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cdot\cos x}$$

✓ ✓

$$= \frac{1+1+2\sin x}{(1+\sin x)\cdot\cos x} = \frac{2+2\sin x}{(1+\sin x)\cdot\cos x} = \frac{2\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}\cdot\cos x}$$

$$= \frac{2}{\cos x} = 2\cdot\frac{1}{\cos x} = \boxed{2\sec x}$$

Cross-Multiply to verify that

$$\frac{1-\sec x}{1+\sec x} = \frac{\cos x - 1}{\cos x + 1}$$

$$(1-\sec x)(\cos x + 1) = (1+\sec x)(\cos x - 1)$$

$$\cos x \cancel{+1} - \underbrace{\cancel{\sec x \cos x}}_{1} - \sec x = \cos x \cancel{-1} + \underbrace{\cancel{\sec x \cos x}}_{1} - \sec x$$

$$\cos x - \sec x = \cos x - \sec x$$

$\sec x = \dfrac{1}{\cos x}$ multiply by $\cos x$

$\cos x \sec x = \cancel{\cos x} \cdot \dfrac{1}{\cancel{\cos x}}$ $\quad \cos x \sec x = 1$ ✓

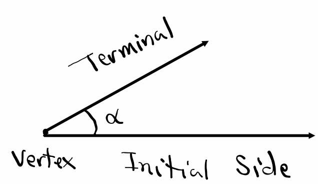
$\sin x \cdot \csc x = 1$

$\tan x \cdot \cot x = 1$

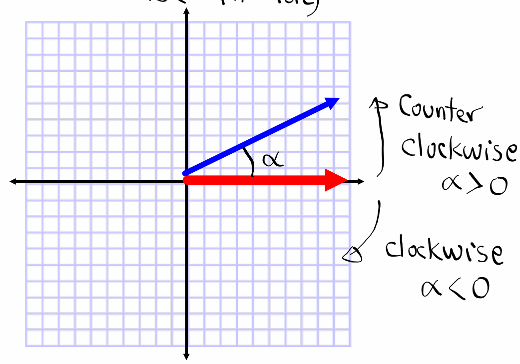
Angles, standard Position

Positive & Negative angles,

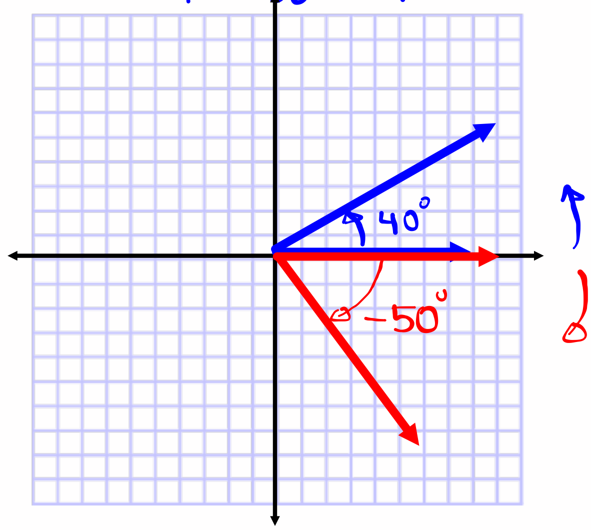
Reference angle.



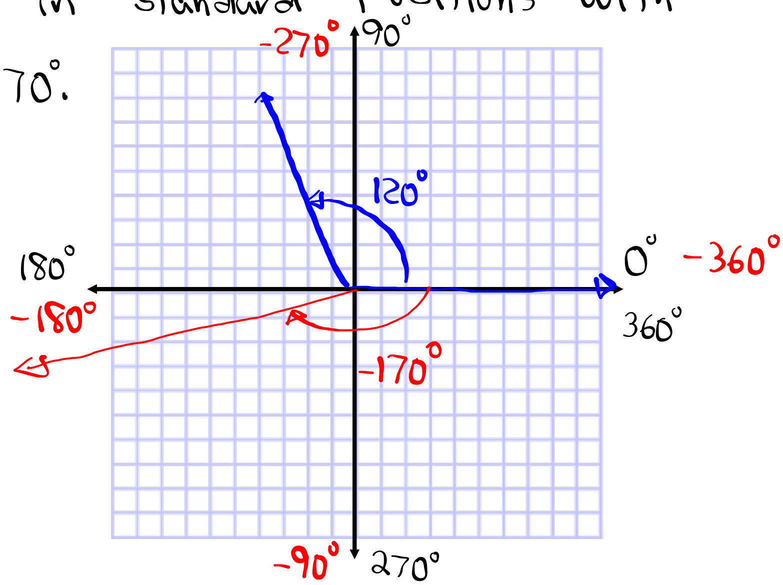
Origin on x-axis (Right Side)



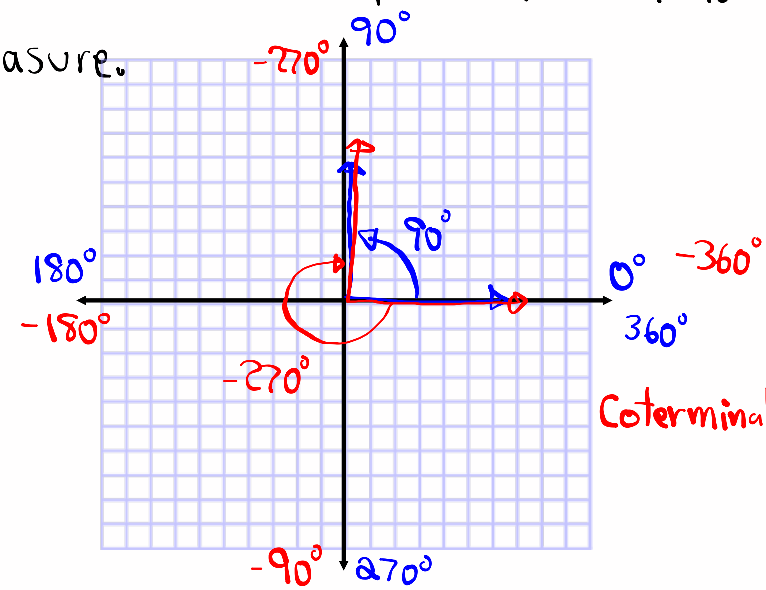
Draw 40° & −50° in standard position.



Draw angles in standard positions with 120° and −170°.



Draw angles in standard position with 90° and -270° measure.



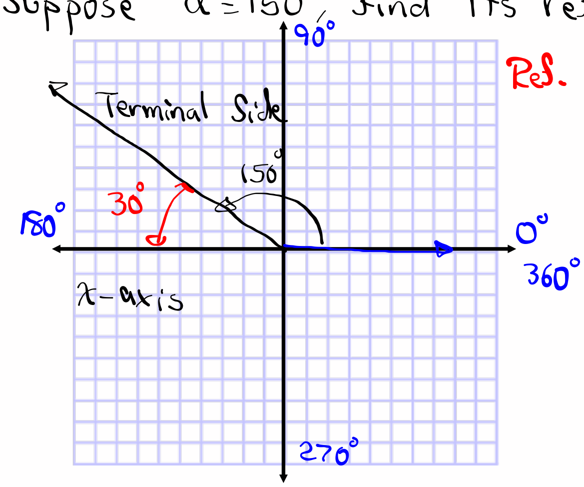
Coterminals

Reference angle:

It is the acute angle that terminal side makes with the x-axis.

$0° < \text{acute angle} < 90°$

Suppose $\alpha = 150°$, find its reference angle.

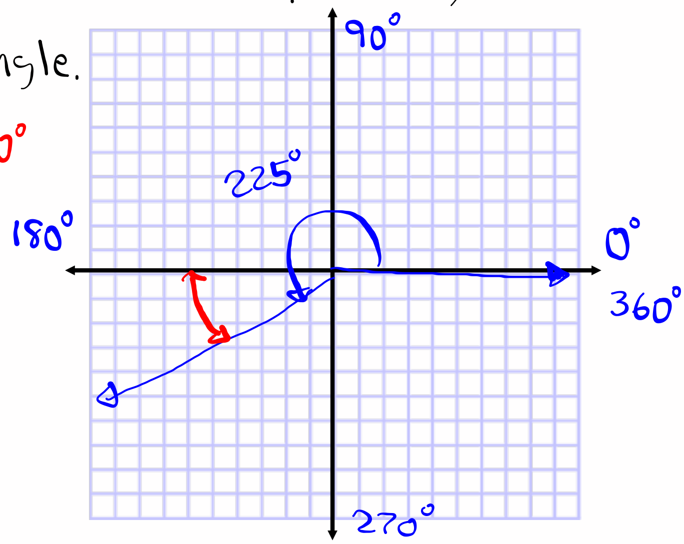


Ref. Angle = $180° - 150° = 30°$

Draw $\alpha = 225°$ in standard position, then find its reference angle.

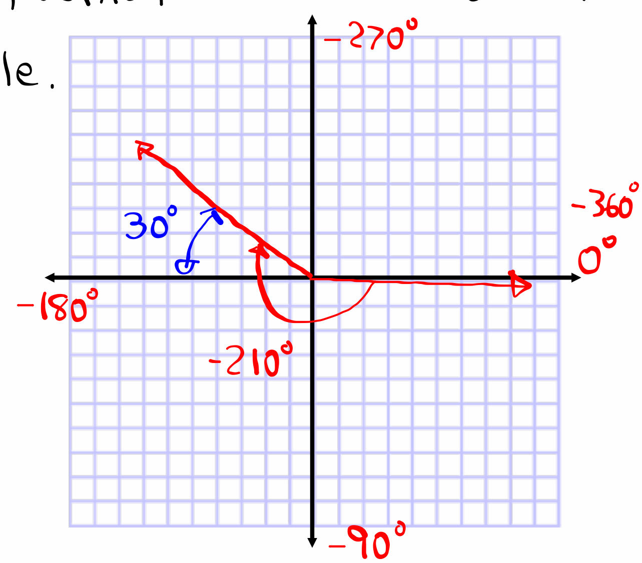
Ref. Angle $= 225° - 180°$

$= 45°$



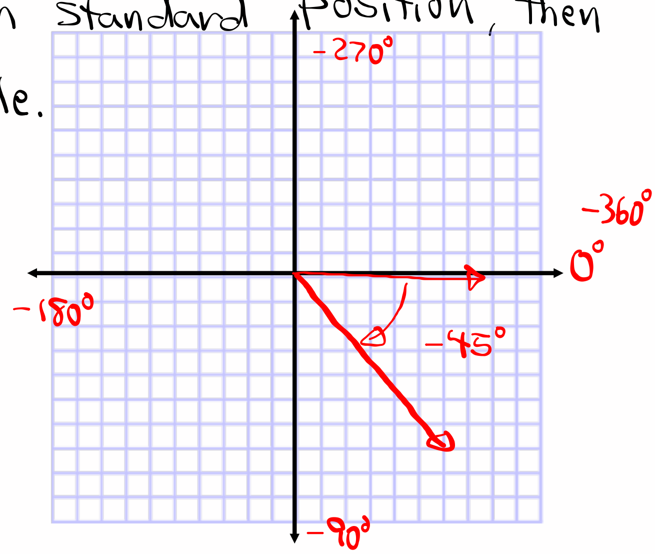
Draw in standard position for $\alpha = -210°$ and find its Ref. angle.

Ref. Angle $= 30°$



Draw $\alpha = -45^\circ$ in standard position, then find its Ref. angle.

clockwise



Ref. Angle $= 45^\circ$

How to find the ref. angle when terminal side is in

1) Q I

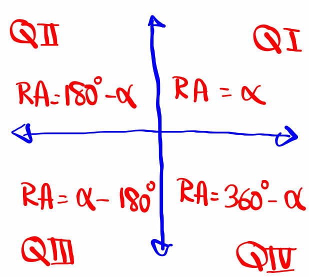
Ref. Angle $= \alpha$

2) Q II

Ref. Angle $= 180^\circ - \alpha$

3) Q III Ref. Angle $= \alpha - 180^\circ$

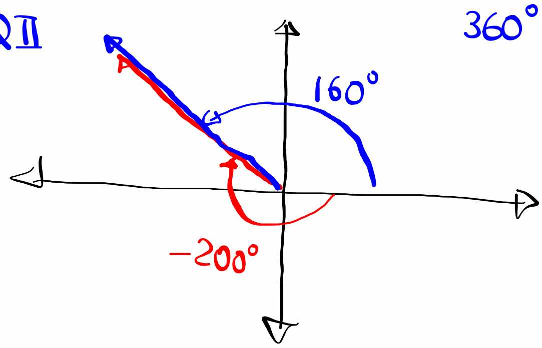
4) Q IV Ref. angle $= 360^\circ - \alpha$



when $\alpha < 0$, find a positive angle with same terminal side.

Find the ref. angle with $\alpha = -200^\circ$

QII



$360^\circ - 200^\circ = 160^\circ$

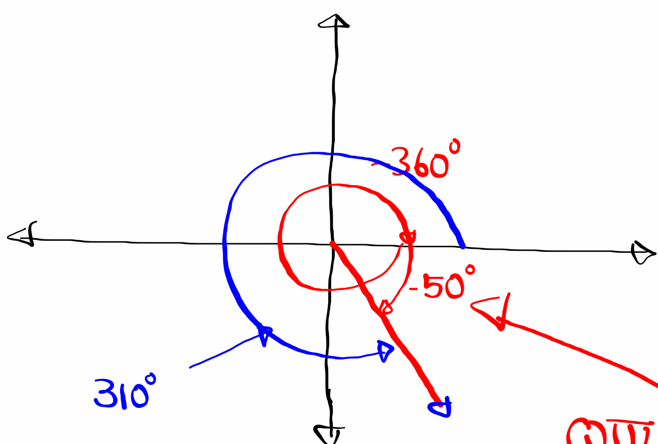
Ref. Angle in QII

$= 180^\circ - 160^\circ$

$= 20^\circ$

Find the ref. angle for $\alpha = -410^\circ$.

$\alpha = -(360^\circ + 50^\circ)$



Coterminal angle

$360^\circ - 50^\circ = 310^\circ$

Ref. Ange $= 360^\circ - \alpha$

$= 360^\circ - 310^\circ$

$= \boxed{50^\circ}$

QIV

Measuring angles in Radians.

$180^\circ = \pi$ Radians

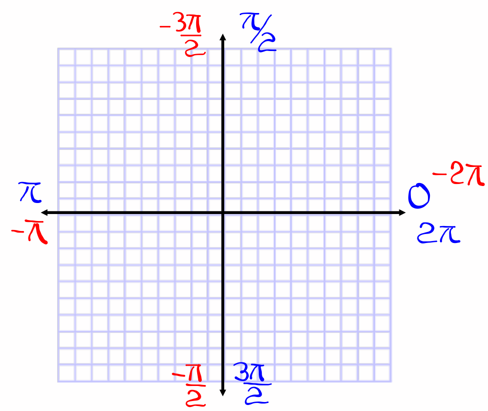
$1^\circ = \frac{\pi}{180}$ Radian

$\frac{180}{\pi}$ degrees $= 1$ Rad.

$60^\circ = \frac{\pi}{3}$

$45^\circ = \frac{\pi}{4}$

$30^\circ = \frac{\pi}{6}$



Convert 75° in radians.

$$75^\circ = 30^\circ + 45^\circ$$

$$= \frac{\pi}{6} + \frac{\pi}{4} = \frac{\pi \cdot 2}{6 \cdot 2} + \frac{\pi \cdot 3}{4 \cdot 3} = \frac{2\pi}{12} + \frac{3\pi}{12} = \frac{5\pi}{12}$$

LCD=12

$$180^\circ = \pi$$

$$1^\circ = \frac{\pi}{180}$$

multiply by 75

$$75^\circ = \frac{\cancel{75}\pi}{180} = \frac{\cancel{15}\pi}{\cancel{36}} = \frac{5\pi}{12}$$

Convert 225° to radians.

$225° = 180° + 45°$

$= \pi + \frac{\pi}{4} = \frac{4\pi}{4} + \frac{\pi}{4} = \boxed{\frac{5\pi}{4}}$

Convert 300° to radians.

$300° = 360° - 60°$

$= 2\pi - \frac{\pi}{3} = \frac{2\pi\cdot 3}{3} - \frac{\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$

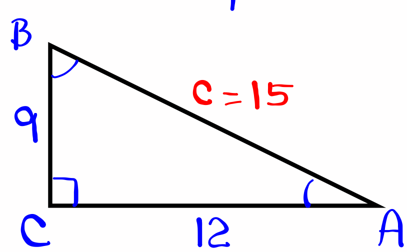
$300° = 270° + 30°$

$= \frac{3\pi\cdot 3}{2\cdot 3} + \frac{\pi}{6} = \frac{9\pi}{6} + \frac{\pi}{6} = \frac{\cancel{10}^{\,5}\pi}{\cancel{6}_{\,3}} = \frac{5\pi}{3}$

Class QZ 3

Use the right triangle below to complete the following chart.

$\sin A = \frac{3}{5}$	$\csc A = \frac{5}{3}$
$\cos A = \frac{4}{5}$	$\sec A = \frac{5}{4}$
$\tan A = \frac{3}{4}$	$\cot A = \frac{4}{3}$



$c^2 = 9^2 + 12^2$

$c^2 = 225$ $\boxed{c = 15}$